Journal of Fluids and Structures 18 (2003) 149–163

JOURNAL OF FLUIDS AND STRUCTURES

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An excitation spectrum criterion for the vibration-induced fatigue of small bore pipes

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Received 6 January 2003; accepted 31 July 2003

Abstract

The purpose of the study is to determine an easy-to-use criterion to evaluate the risk of vibration-induced fatigue of small bore pipes. The failure mechanism considered is the resonant amplification of a stationary broadband excitation by the main pipe, leading to bending stresses above the fatigue limit of the steel. Based on the Euler beam theory, a simple model is built up for the natural mode shapes of the small bore pipe close to its root. It is shown that the velocity spectrum at the root of the small bore pipe is equal to the r.m.s. value of the bending stress multiplied by a function of the natural frequency, the damping coefficient, the speed of elastic waves in the steel, Young's modulus and a nondimensional factor weakly depending on the geometry of the small bore pipe. A maximum velocity spectrum can then be deduced, assuming that a small bore pipe vibrates mainly on its natural mode shapes. The maximum excitation spectrum is defined for each frequency f as the one which would generate a maximum bending stress equal to the endurance limit of the steel, would the small bore pipe have a natural frequency equal to f. Using envelope values of the nondimensional factor, the stress intensification factor, the peak factor and the endurance limit of the steel, one obtains the following maximum velocity spectrum for the stainless steel: $v < 6\,\mathrm{mm/s}/\sqrt{f}$, and the following maximum velocity spectrum for the ferritic steel: $v < 2.7\,\mathrm{mm/s}/\sqrt{f}$. The velocity spectrum criterion appears less penalizing than the 12 mm/s criterion and more conservative than the strict enforcement of the ANSI-OM3 standard. Comparisons with former plant studies show that the velocity spectrum criterion leads to the correct fatigue diagnosis.

1. Introduction

Several failures of small bore piping systems have occurred in French nuclear power plants because of fatigue due to flow-induced vibrations, as described by Moussou and Boyelle (1999) (Moussou et al., 2001). Evaluation of the risk of fatigue failure is thus required for small bore pipes in steady operating conditions.

Expert methods for the vibration of small bore pipes such as modal analysis and operating deflection shapes analysis are well established, as indicated by Richardon (1997). The control of small bore pipes continues to be an issue because of their large number in safety related systems. Time-consuming methods requiring expertise cannot be used, and simple criteria are needed to optimize control operations.

In structural mechanics terms, the difference between main pipes and small bore pipe vibration is that the excitation sources of small bore pipes are forced displacements, with a rather complicated spectrum such as the one shown in Fig. 1. As can be seen, the excitation spectrum of a small bore pipe can exhibit a significant level up to 400 Hz, with several modes.

The regulation for vibration of piping systems is given by ANSI-OM3 (1982). This is based on the theory of vibrating beams, and stipulates that if the r.m.s. velocity everywhere on a pipe is lower than a prescribed value, dependent on the

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doi:10.1016/j.jfluidstructs.2003.07.002

Nomenclature

C_1	nondimensional coefficient of the ANSI-OM3, describing the influence of concentrated masses and varying from 0.25 to 1
C_3	nondimensional coefficient of the ANSI-OM3, describing the added mass factor (water), varying from 1 to about 1.3 for small bore pipes
C_4	nondimensional coefficient of the ANSI-OM3, describing the velocity to stress ratio as a function of the layout and varying from 0.7 to 1.33
c_{steel}	speed of the compression waves in steel, equal to $(E/\rho_{steel})^{1/2}$: 5000 m/s for current steel.
E	Young modulus of the steel (MPa)
f	frequency (Hz)
f_n	natural frequency of the small bore pipe (Hz)
F	shear force spectrum in the small bore pipe $(\mathrm{N}/\sqrt{\mathrm{Hz}})$
F_{peak}	peak factor: ratio of the maximum value to the r.m.s. value of the stress, equal to 3.5
$G(u)$	nondimensional elastic energy in the root of the small bore pipe, equal to: $G(u) = u + \frac{1}{2}\cos 2u - 2\mathrm{e}^{-u}\sin u - \frac{1}{2}\mathrm{e}^{-2u}$
i	stress intensification factor, equal to 2.1 for a socket welding and 1.8 for a butt welding
I	main pipe inertia, equal to $(\pi/4)(R_{out}^4 - R_{in}^4)$ (m^4)
K	coefficient equal to $E/\alpha c_{steel}$ (MPa s/m)
k	bending wavenumber (m^{-1}), defined by: $EIk^4 = \lambda\omega^2$
L	straight length of the small bore pipe (m)
l_v	length of the valve (m)
M	bending moment spectrum in the small bore pipe $(\mathrm{N/m}\sqrt{\mathrm{Hz}})$
R_{out}	outer radius of the small bore pipe (m)
R_{in}	inner radius of the small bore pipe (m)
u	nondimensional space coordinate, defined as $u = kx$
V_{all}	allowable r.m.s. velocity according to the ANSI-OM3 standard
v_{max}	maximum velocity spectrum in the small bore pipe $(\mathrm{m/s}\sqrt{\mathrm{Hz}})$
v_{root}	velocity spectrum at the root of the small bore pipe $(\mathrm{m/s}\sqrt{\mathrm{Hz}})$
x	abscissa along the small bore pipe (m)
y	bending deflection spectrum along the small bore pipe $(\mathrm{m}/\sqrt{\mathrm{Hz}})$
α	nondimensional factor, varying from 0.2 to 0.3 for usual pipes, defined as: $\alpha = \frac{1}{2}\sqrt{\rho_{steel} I/(\lambda\, R_{out}^2)}$
η	structural damping of the small bore pipe
λ	mass per unit length of the small bore pipe, equal to: $\pi\rho_{steel}(R_{out}^2 - R_{in}^2) + \pi\rho_{water}R_{in}^2$ (kg/m)
ρ_{steel}	steel density, equal to $7800\,\mathrm{kg/m^3}$
ρ_{water}	water density, equal to $1000\,\mathrm{kg/m^3}$
σ_{lim}	fatigue limit of the steel (at 10^{11} cycles), equal to 52 MPa for ferritic steel and 114 MPa for austenitic steel
σ_0	beam bending stress spectrum at the root of the small bore pipe $(\mathrm{MPa}/\sqrt{\mathrm{Hz}})$
ω	pulsation (rad/s), equal to $2\pi f$

pipe parameters, there is no risk of fatigue failure (Wachel, 1995). In a similar approach, a widespread rule of thumb stipulates that a pipe which vibrates below 12 mm/s is not vulnerable to vibration-induced fatigue.

The ANSI-OM 3 standard does not give guidance for determining the location of the maximum velocity on complicated small bore pipes, such as the one reproduced in Fig. 2. The on-site technician has to decide where the small bore pipe has a velocity maximum. This operation can be time-consuming: for instance, the small bore pipe reproduced in Fig. 2 would require at least six to seven points of measurement in three perpendicular directions in order to determine the location where the velocity is maximum. In the case of a nuclear power plant, such an operation may be difficult to achieve whenever a highly radioactive environment surrounds the small bore pipe.

Furthermore, an r.m.s. velocity measurement on a small bore pipe, although providing a reasonably fair estimation of the stress, cannot provide information about the spectral content of the vibration. A broadband vibration of the main pipe due to valve cavitation could generate a small bore pipe vibration mode similar to that due to the blade passing frequency of a pump. Yet, in the first case, the small bore pipe would be exposed to vibration-induced fatigue whatever its natural frequencies, while in the second case, the addition of a small mass would be enough to decrease its vibration by a factor of 10.

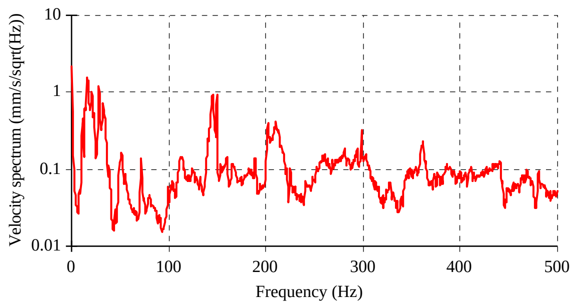

Fig. 1. Typical velocity spectrum (square root of the PSD) at the root of a small bore pipe.

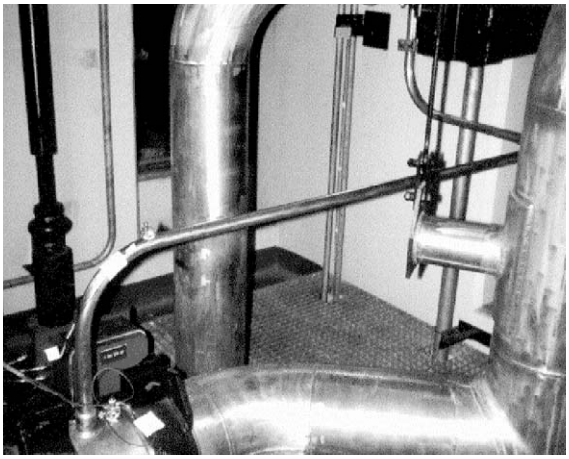

Fig. 2. Complicated small bore pipe layout in a safety piping system.

Finally, a velocity of 5 mm/s measured on a small bore pipe could hide a risk of coincidence with a main pipe natural frequency: a small change in the operating conditions could be enough to generate a high vibration level in the small bore pipe.

In summary, three reasons for developing a criterion based on the spectrum of the excitation can be given; the first reason is that a single measurement at a predefined position is easy to specify and obtain. The second reason is that the velocity spectrum at the root of the small bore pipe can be related to operating conditions, whereas the maximum r.m.s. velocity of the small bore pipe can hardly be interpreted. The third reason is that r.m.s. measurement according to the ANSI-OM 3 standard may hide the risk of a coincidence between a main pipe mode and a small bore pipe mode.

2. Scope of the criterion

The criterion presented hereafter deals with the fatigue risk evaluation of a small bore pipe in a given set of operating conditions. The input data of this evaluation are the velocity measurements in the operating conditions which are assumed to be the most damaging for the small bore pipe; how to determine these damaging conditions is beyond the scope of the present paper.

The proposed criterion is to be used as a screening one: if a small bore pipe excitation remains below the threshold spectrum, there is no risk of fatigue failure. If the excitation exceeds the threshold, there might be a risk, and further evaluation is required to determine whether corrective actions should be undertaken.

The failure mechanism considered is the resonant amplification of the main pipe vibrations. No other source of excitation is considered.

The vibrations of the main pipe are assumed to be a stationary broadband process. The vibrations due to a pure harmonic excitation are not investigated; other techniques have been proposed by Von Nimitz (1974) in this case. Neither transient processes nor static deformations are investigated.

3. Calculations

When submitted to stationary flow-induced vibrations, a small bore pipe vibrates mainly on its natural modes. The vibration level can be harmless, provided that the excitation, i.e., the main pipe displacement, remains low enough. The issue is thus to determine the maximum velocity spectrum that a small bore pipe can tolerate at its root, in the range of frequencies close to its natural frequency.

The calculation steps are the following:

(i) determination of the mode shape of the small bore pipe,
(ii) determination of the deflection amplitude as a function of the excitation spectrum, based on the fact that the dissipation due to the damping balances the power provided by the main pipe movements,
(iii) determination of the r.m.s. stress as a function of the excitation spectrum,
(iv) elaboration of a criterion using the expression of the r.m.s. stress, the peak factor and the stress intensification factor.

3.1. Hypotheses and assumptions

The small bore pipe is modelled as shown in Fig. 3. It is assumed that the small bore pipe excitation is a lateral displacement imposed at its root, and that the main part of the elastic energy of the small bore pipe is localized within a straight part of length L starting from its root. Furthermore, it is assumed that the deflection shape of the small bore pipe is similar to a mode shape, and that the main pipe compliance is equal to zero, so that the angle of deflection of the small bore pipe is zero at its root. The Tresca stress is therefore assumed maximal at the root of the small bore pipe, and equal to the bending stress, i.e., the contribution of torsion to the total stress is assumed negligible.

The spectra used in the present paper are defined as the square roots of the PSD, with a scale factor making the area of the PSD equal to the mean square value of the time history. Square roots are used instead of PSD because harmonic calculation becomes easier: for instance, the stress spectrum is equal to the excitation spectrum multiplied by the transfer function.

The velocity spectrum and the bending moment are basically complex spectra. As all physical values are to be expressed as functions of σ_0, this latter spectrum is chosen real, and all other spectra have their phase referenced to it. For the sake of simplicity, the modulus symbol is omitted in the following sections when no confusion can occur.

3.2. Mode shape of the small bore pipe

As indicated, the deflection shape of the straight part of the small bore pipe is assumed to be almost identical to a mode shape, i.e., a shape with a clamp condition on the left side. The main pipe movement is neglected in this part of the study.

It is shown in Appendix A that for a given frequency, the mode shape of the straight pipe complying with a clamp condition on the left is

$$\frac{v(x)}{c_{\text{steel}}} = \alpha \frac{\sigma_0}{E} [\exp(-kx) - \cos(kx) + \sin(kx)] \tag{1}$$

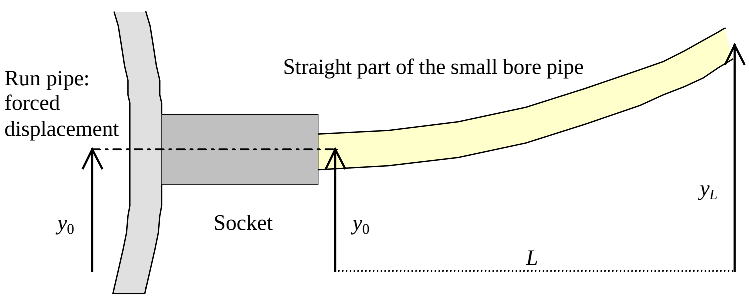

Fig. 3. Small bore pipe schematic drawing.

and the bending moment is expressed as

$$M(x) = \frac{I\sigma_0}{2R_{\text{out}}}[\exp(-kx) + \cos(kx) - \sin(kx)], \tag{2}$$

where c_{steel} is the speed of sound in the steel, α is a nondimensional parameter, varying from 0.2 to 0.3, σ_0 is the bending stress spectrum at the root of the small bore pipe and k is related to the pulsation ω by the dispersion equation

$$k^2 = \frac{\omega}{2\alpha c_{\text{steel}} R_{\text{out}}}.$$

The deflection shape and the bending moment of the straight part of the small bore pipe are shown in Fig. 4. Expressions (1) and (2) are assumed to be valid for a wide range of boundary conditions, as shown in Figs. 5–7.

In the case of an elbow, i.e., a very flexible element, a zero moment condition can be assumed, so that $kL \simeq 1$ (see Fig. 5). In the case of a valve with a smaller length l_v than the tube's one, the bending moment at the root of the small bore pipe is only due to the valve acceleration, and $k(L + l_v) \simeq 1$ (see Fig. 6). In the case of a support, a clamp or a

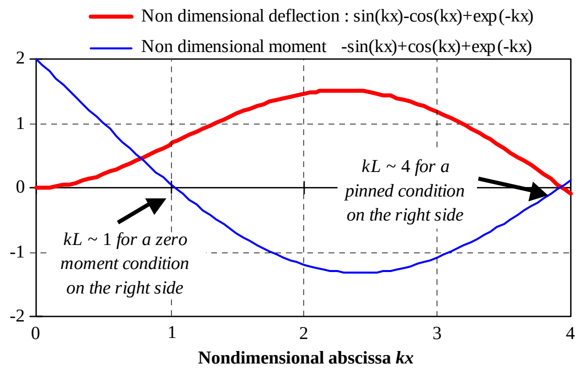

Fig. 4. Small bore pipe deflection and moment.

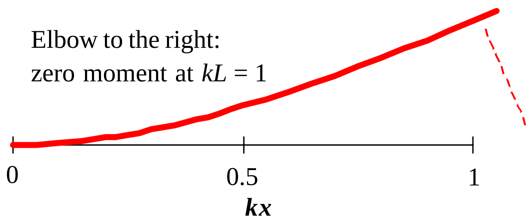

Fig. 5. Mode shape for an elbow on the right side.

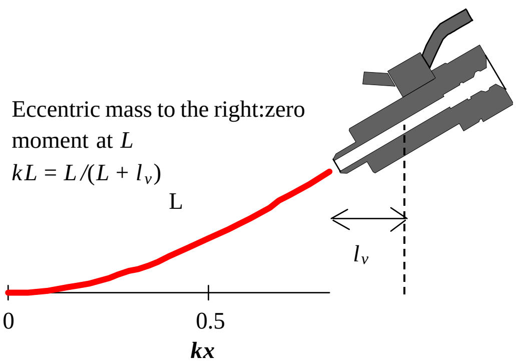

Fig. 6. Mode shape for a valve on the right side.

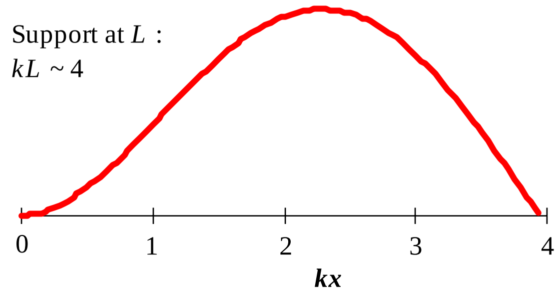

Fig. 7. Mode shape for a support on the right side.

pin condition is generated, and kL is equal to 4 (see Fig. 7). In all the aforementioned situations the kL term is higher than 0.5. This relation holds for the first natural mode, and kL is of course higher for higher modes.

3.3. Energy balance of the deflection shape

The structural damping of the small bore pipe is assumed small (typically lower than 1%) so that the velocity at the root is much smaller than the velocity at distance L. The shear force spectrum at the root is thus almost equal to its modal value

$$F_0 = \frac{\sigma_0 I k}{R_{\text{out}}}.$$

The rate of energy flow to the small bore pipe P_{in} is equal to the shear force F_0 multiplied by the velocity v_{root} at the base. Using complex notations, for a given wavenumber k and for a given frequency f, the rate of energy flow is

$$P_{\text{in}} = \frac{1}{2}\,\text{Re}(F_0^* v_{\text{root}}) = \frac{\sigma_0 I k}{2R_{\text{out}}}|v_{\text{root}}|.$$

The rate of dissipation P_{dis} due to damping can be estimated using a mass-spring analogy as in Gibert (1988). It is shown that the dissipation of energy during one cycle is related to the total energy U_t and the structural damping coefficient η by

energy dissipated $= 4\pi\eta$ total energy.

The rate of dissipation P_{dis} is equal to the energy dissipated during one cycle divided by the duration of the cycle

$$P_{\text{dis}} = 4\pi\eta f U_t.$$

An estimation of the total energy of the small bore pipe is now needed. It is well known that the total energy is equal to the double of the elastic energy [see Landau and Lifchitz (1974) for instance] for the natural modes of a linear oscillator. Assuming that all the elastic energy of the small pipe is due to the bending of the straight pipe of length L, one gets

$$U_t = \int_0^L \frac{M^2}{EI}\,\text{d}x = \frac{1}{k}\int_0^1 \frac{M^2}{EI}\,\text{d}(kx) = \frac{1}{k}\frac{1}{EI}\left(\frac{\sigma_0 I}{2R_{\text{out}}}\right)^2 G(kL),$$

where $G(kL) = \int_0^{kL}[\exp(-u) + \cos(u) - \sin(u)]^2\,\text{d}u$ can be solved analytically (see Nomenclature for its expression). As can be seen in Fig. 8, G varies from 1.35 to 4 for kL varying from 0.5 to 4. The dissipated power can then be expressed as

$$P_{\text{dis}} = \frac{4\pi\eta f}{k}\frac{1}{EI}\left(\frac{\sigma_0 I}{2R_{\text{out}}}\right)^2 G(kL).$$

In steady regime, the rate of energy flow balances the dissipation. By equating P_{in} to P_{dis} and dropping the modulus symbol, one gets

$$\frac{\eta\omega}{k^2}\frac{\sigma_0}{ER_{\text{out}}}G(kL) = v_{\text{root}}.$$

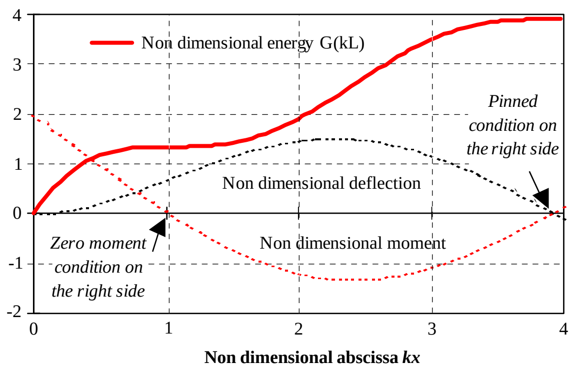

Fig. 8. Nondimensional energy of the small bore pipe.

The term ω/k^2 is removed using the dispersion relation

$$2\alpha G(kL)\frac{\sigma_0}{E} = \frac{v_{\mathrm{root}}}{\eta c_{\mathrm{steel}}}. \tag{3}$$

The above equation contains the fatigue criterion in a nutshell: the relation between the bending stress and the forced velocity only depends on the damping, on two factors α and G which are roughly constant and on physical constants. Velocity appears as a natural estimator of the bending stress; this feature is the base of velocity-based criterion for the control of piping system vibrations proposed by Wachel (1995) and Karczub and Norton (1999).

3.4. R.m.s. stress due to a broadband excitation

Eq. (2) links the root velocity spectrum to the root bending stress spectrum. In order to get an estimation of the physical stress, i.e., a stress expressed in MPa, an integration of the bending stress spectrum in the frequency range close to the natural frequency f_n of the small bore pipe has to be performed. Let the excitation spectrum $v_{\mathrm{root}}(f)$ be roughly constant for the frequencies close to the natural frequencies. Using again a mass–spring analogy, it is easily shown that for a frequency close to a natural frequency f_n, the stress spectra is expressed as a function of the velocity spectra:

$$\sigma_0(f) = \frac{K v_{\mathrm{root}}(f_n)}{(1-(f^2/f_n^2)) + 2\mathrm{j}\eta},$$

K being a constant, which can be determined using the Eq. (2) for $f = f_n$ and $\mathrm{j} = \sqrt{-1}$. Then,

$$\sigma_0(f) = \frac{E}{\alpha G(kL) c_{\mathrm{steel}}} \frac{v_{\mathrm{root}}(f_n)}{(1-(f^2/f_n^2)) + 2\mathrm{j}\eta}$$

For small values of the damping coefficient, most of the r.m.s. stress is due to the frequencies close to f_n. Integrating the square modulus of the above expression, one gets

$$\sigma_0^{\mathrm{r.m.s.}} \approx \frac{E v_{\mathrm{root}}(f_n)}{\alpha G(kL) c_{\mathrm{steel}}} \sqrt{\int \frac{\mathrm{d}f}{(1-(f^2/f_n^2))^2 + 4\eta^2}}.$$

A fair approximation of the above integral is $\pi f_n/4\eta$, so that

$$\sigma_0^{\mathrm{r.m.s.}} \approx \frac{E v_{\mathrm{root}}(f_n)}{\alpha G(kL) c_{\mathrm{steel}}} \sqrt{\frac{\pi}{\eta}} \frac{\sqrt{f_n}}{2}.$$

The r.m.s. stress is related to the root velocity spectrum by a factor depending upon the damping coefficient, upon the factors G and α which are roughly constant and upon the square root of the frequency. This last dependency appears self-evident according to dimensional considerations.

3.5. Fatigue criterion expression

The expected lifetime of a power plant being of several tens of years, and the natural frequencies of small bore pipes ranging from 10 to 200 Hz, the evaluation of the failure risk should be based on high-cycle fatigue. Following the same approach as in the ANSI-OM3 standard, the fatigue criterion for the r.m.s. beam stress at the root of the small bore pipe is defined by

$$\sigma_0^{\text{r.m.s.}} \leqslant \frac{\sigma_{\text{lim}}}{iF_{\text{peak}}},$$

i being the stress intensification factor (2.1 for a socket welding and 1.8 for a butt welding), F_{peak} being the ratio of the temporal maximal stress to the r.m.s. stress, conventionally taken equal to 3.5 (see Ibrahim, 1985; Sobczyk and Spencer, 1992), σ_{lim} being the fatigue limit of steel: the B-curve of the ASME tables indicates 114 MPa for stainless steel and 52 MPa for austenitic steel for 10^{11} cycles.

The fatigue criterion is therefore easily expressed for the root velocity as

$$v_{\text{root}}(f_n) \leqslant \frac{1}{\sqrt{f_n}} \frac{2\alpha\sqrt{\eta}G(kL)}{\sqrt{\pi}} \frac{c_{\text{steel}}}{E} \frac{\sigma_{\text{lim}}}{iF_{\text{peak}}}. \tag{4}$$

Let a general fatigue criterion be now defined, based on the above expression. The threshold spectrum should exhibit the following property: if the velocity spectrum at the root of the small bore pipe is lower than the threshold, there is no risk of vibration-induced fatigue failure. According to the preceding concepts, fatigue can occur if the velocity spectrum exceeds expression (4) at a natural frequency of the small bore pipe. In a penalizing approach, the worst situation would occur when two natural frequencies are simultaneously involved. A screening criterion can thus be proposed by dividing expression (4) by a factor $\sqrt{2}$ so that the quadratic summation of the stresses associated with the two natural frequencies does not exceed the fatigue limit of the steel.

A further step is needed to define a layout-independent criterion. This is achieved by demanding the root velocity to be lower than expression (4) divided by the factor $\sqrt{2}$ for any value of the frequency f_n. Hence, whatever its natural frequencies, a small bore pipe is not exposed to fatigue failure if the root velocity spectrum fulfills the following condition:

$$v_{\text{root}}(f) \leqslant \frac{1}{\sqrt{f}} \frac{2\alpha\sqrt{\eta}\,G(kL)}{\sqrt{\pi}} \frac{c_{\text{steel}}}{E} \frac{\sigma_{\text{lim}}}{iF_{\text{peak}}}. \tag{5}$$

Eq. (5) is a screening fatigue criterion for small bore pipes, expressed using root velocity spectrum measurements. Suggested values of the coefficients are given in Table 1 for socket-welding small bore pipes. Using these values, the criterion for stainless steel can be written as

$$v_{\text{root}}(f) \leqslant \frac{6\ \text{mm/s}}{\sqrt{f}}$$

and for the ferritic steel as

$$v_{\text{root}}(f) \leqslant \frac{2.7\ \text{mm/s}}{\sqrt{f}}.$$

Table 1
Proposed coefficients of the criterion

Parameters	Ferritic steel	Stainless steel	Units
η	5×10^{-3}	5×10^{-3}	
c_{steel}	5000	5000	m/s
E	2×10^{5}	2×10^{5}	MPa
α	0.2	0.2	
$G(kL)$	1.35	1.35	
σ_{lim}	52	114	MPa
i	2.1	2.1	
F_{peak}	3.5	3.5	
$v_{\text{root}}(f)\ \sqrt{f}$	2.7	5.9	mm/s

4. How to use the velocity spectrum criterion

The purpose of the velocity spectrum criterion is to easily identify small bore pipes which are not exposed to vibration-induced fatigue.

It should be used the following way: if the root velocity spectrum is lower than the criterion, the small bore pipe is safe. If the root velocity spectrum crosses the threshold, the small bore pipe might be exposed to vibration-induced fatigue and further investigation is required. In this case, the risk depends on the frequency for which the excitation crosses the threshold: the closer to a natural frequency, the higher the risk.

It must be highlighted that the criterion should be used for broadband vibrations, i.e., for peaks broader than a typical small bore pipe resonance. The criterion cannot be used for narrow excitations such as an excitation due to a pump passing blade frequency. It is the author's belief that a criterion for narrow excitations would be useless, because the current peaks would generally be too high. Coincidence between the natural frequencies of the small bore pipes and the pump passing blade frequency and its harmonics should be avoided.

5. Comparison with a maximum deflection approach

In the present section, the velocity spectrum criterion is compared to deflection-based criteria. The reference values of the stress and the deflection are assumed to be correctly given by Eqs. (1) and (3).

5.1. ANSI-OM3 applied to small bore pipes

An analysis of the ANSI-OM3 criteria was given by Baratte et al. (1998). Applied to small bore pipes, the ANSI-OM3 allowable r.m.s. velocity can be rewritten using the present paper's notations:

$$V_{\text{all}} = \sigma_{\text{lim}} \frac{C_1 C_4}{F_{\text{peak}} C_3} \frac{13.4\,\text{mm}/(\text{s MPa})}{i},$$

where C_1 varies from 0.25 to 1 and stands for the influence of the concentrated masses, C_3 varies from 1 to 1.3 and stands for the added mass due to water, C_4 varies from 0.7 to 1.33 and is a layout-dependent velocity to stress ratio.

5.2. Case of a straight pipe with a small heavy valve

Let a straight pipe with a valve at its end vibrate on its first natural mode. The valve is small compared to the length of the tube, the diameter of the tube is 1 in (2.54 mm) and the structural damping coefficient is equal to 1%; α is equal to 2.8, the natural frequency f_n is associated to $kL = 1$ (see Fig. 6 for $l_v \ll L$) so that $G(kL) = 1.5$ (see Fig. 8), and the maximum bending stress is equal to the fatigue limit if (Eq. (3))

$$v_{\text{root}}(f_n)\sqrt{f_n} = \frac{2 \times 0.28 \times \sqrt{0.01} \times 1.35}{\sqrt{\pi}} \frac{5000}{2 \times 10^{11}} \frac{114}{2.1 \times 3.5} = 16.5\,\text{mm/s}.$$

There is a margin factor of 2.8 between the actual fatigue threshold and the $6\,\text{mm/s}/\sqrt{f}$ criterion.

The application of the ANSI-OM3 criterion provides the following value of the maximum velocity:

$$V_{\text{all}} = 114 \frac{0.25 \times 0.7}{3.5 \times 1.2} \frac{13.4}{2.1} = 30\,\text{mm/s}.$$

For $kL = 1$, the nondimensional deflection shape has a maximum value equal to 0.7 (see Fig. 4) and the maximum r.m.s. velocity is actually given by

$$V_{\text{all}} = \alpha\, c_{\text{steel}} \frac{\sigma_{\text{lim}}}{iF_{\text{peak}}} \frac{1}{E} 0.7 = 0.28 \times 5000 \frac{114 \times 10^6}{2.1 \times 3.5} \frac{0.7}{2 \times 10^{11}} = 76\,\text{mm/s}.$$

There is a margin factor of 2.5 between the actual fatigue threshold and the ANSI-OM3 criteria. The margin factor of the 12 mm/s is equal to 6.3.

5.3. Case of a straight pipe with a pinned condition

Let a straight pipe without any valve or mass and a pinned condition at its end vibrate on its first natural mode.

The diameter of the tube is 1 in (2.54 mm) and the structural damping coefficient is equal to 1%. α is equal to 2.8, the natural frequency f_n is associated to $kL = 4$ (see Fig. 7) so that $G(kL) = 4$ (see Fig. 8) and the maximum bending stress is equal to the fatigue limit if

$$v_{\text{root}}(f_n)\sqrt{f_n} = \frac{2 \times 0.28 \times \sqrt{0.01} \times 4}{\sqrt{\pi}} \frac{5000}{2 \times 10^{11}} \frac{114}{2.1 \times 3.5} = 49\,\text{mm/s}.$$

There is a margin factor of 8 between the actual fatigue threshold and the $6\,\text{mm/s}/\sqrt{f}$ criterion.

The application of the ANSI-OM3 criteria provides the following value of the maximum velocity:

$$V_{\text{all}} = 114 \frac{1 \times 0.7}{3.5 \times 1.2} \frac{13.4}{2.1} = 120\,\text{mm/s}.$$

The pinned condition is associated to $kL = 4$, and the nondimensional deflection shape has a maximum value equal to 1.5 (see Fig. 4) and the maximum r.m.s. velocity is actually given by

$$V_{\text{all}} = \alpha c_{\text{steel}} \frac{\sigma_{\text{lim}}}{iF_{\text{peak}}} \frac{1}{E} 0.7 = 0.28 \times 5000 \frac{114 \times 10^6}{2.1 \times 3.5} \frac{1.5}{2 \times 10^{11}} = 163\,\text{mm/s}.$$

There is a margin factor of 1.35 between the actual fatigue threshold and the ANSI-OM3 criteria. The margin factor of the 12 mm/s is equal to 13.5.

The comparison made on both cases shows that the velocity spectrum criterion appears less conservative than the 12 mm/s criterion and more conservative than the strict enforcement of the ANSI-OM3 standard. This should be considered as a trend: the velocity spectrum criterion could be less conservative than the ANSI-OM3 in the case of broadband excitations, when the small bore pipe vibrates on several modes simultaneously.

6. Validation

The velocity spectrum criterion was evaluated using root measurements of six small bore pipes for which a vibration-induced fatigue diagnosis had already been made. These small bore pipes were located on different piping systems from different nuclear plants, with different operating regimes, and they had layouts of all types: straight pipes with a valve as well as curved pipes with supports.

A fair agreement was found between the former diagnosis and the application of the present paper's criterion as shown in the Figs. 9–12, where the bold line is the velocity threshold. As can be seen, the measured velocity spectrum crosses the threshold for all the unsafe small bore pipes.

In Fig. 11, three velocity spectra from two different plants at three different times are plotted. Though the velocity spectra are not identical, the diagnosis is the same in the three cases. In Fig. 12, two velocity spectra measured in two different operating conditions are plotted. One of the operating conditions was safe, the other unsafe.

As mentioned before, the criterion does not hold for narrow excitations such as the one associated with a blade passing frequency. Hence, the narrow peaks crossing the criterion threshold in Figs. 10 and 11 should be disregarded, because a fair design of the small bore pipes should avoid coincidence between the natural frequencies of the pipe and the harmonics of the rotational speed of the pump.

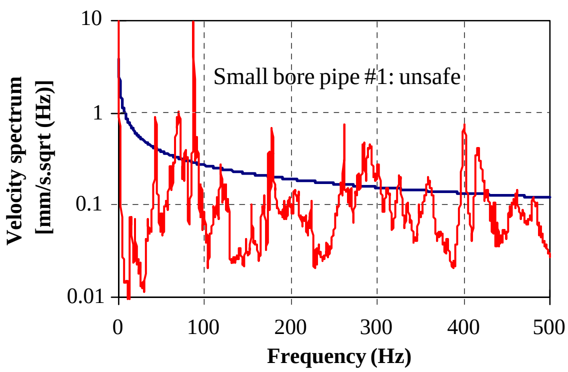

Fig. 9. Root velocity measurement of small bore pipe #1.

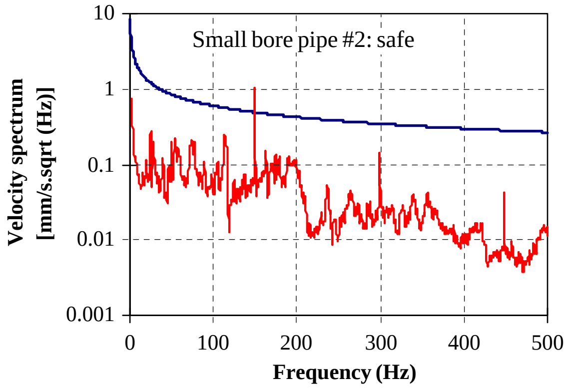

Fig. 10. Root velocity measurement of small bore pipe ♯2.

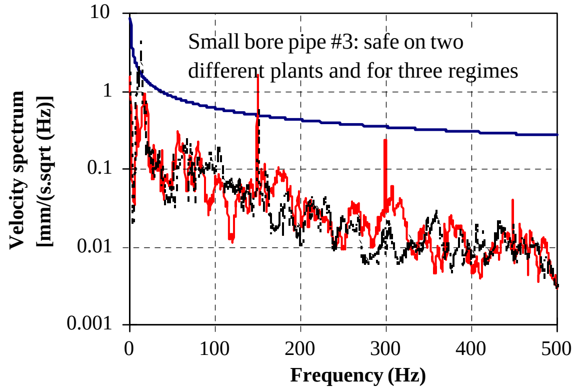

Fig. 11. Root velocity measurement of small bore pipe ♯3 on two different plants.

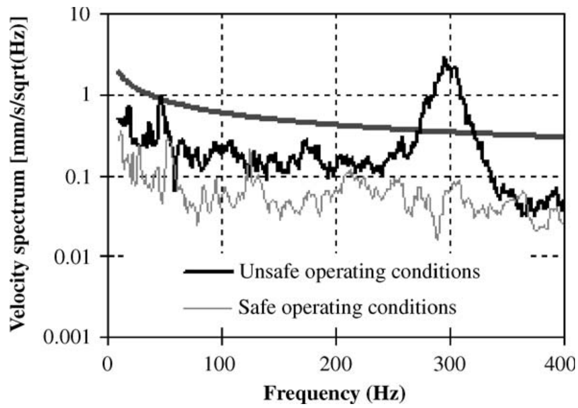

Fig. 12. Root velocity measurement of small bore pipe ♯4 on two different plants.

It should be noted that a comparison with the 12 mm/s and ANSI-OM3 criteria could not be made in Figs. 9–12, because these latter criteria deal with the velocity at the maximum deflection, and not at the base of the small bore pipe.

7. Conclusions

A root velocity criterion has been proposed for the evaluation of the risk of vibration-induced fatigue of small bore pipes in industrial piping systems. It has the following advantages: the location of the measurement is well-defined, and information about a possible coincidence with a main pipe resonance is provided.

The criterion appears less conservative than the 12 mm/s rule of thumb and generally more conservative than the ANSI-OM3 criterion. It is validated by comparison with former analysis of industrial issues.

Acknowledgements

Special thanks are due to Messrs Paulhiac and Guillou for fruitful discussions and field data. Extra thanks are due to Mr Papaconstantinou who kindly took care of the corrections while the author was absent.

Appendix A. Approximate mode shape of a small bore pipe

A.1. General expression of a deflection shape

Let a straight pipe of length L be described by the theory of Euler–Bernoulli beams, as described in Roark (1965). For a given pulsation ω, the deflection shape spectrum is a combination of harmonic and exponential terms

$$y(x) = A\cos(kx) + B\sin(kx) + C\exp(-kx) + D\exp(kx),$$

k being related to the pulsation ω by the dispersion equation

$$EIk^4 = \lambda\omega,$$

λ stands for the mass per unit length of the pipe, equal to

$$\pi\rho_{\text{steel}}(R_{\text{out}}^2 - R_{\text{in}}^2) + \pi\,\rho_{\text{water}}R_{\text{in}}^2.$$

The angle, bending moment and shear force spectra can be deduced from the deflection shape spectrum by

$$\theta(x)/k = \partial y/\partial(kx), \quad M(x)/EIk = \partial y/\partial(kx), \quad -F(x)/EIk = \partial y/\partial(kx).$$

A.2. Mode shape approximation

In this section, an expression of the mode shape of the small bore pipe is derived assuming that the term $\exp(kx)$ is negligible, which is justified later. A straightforward calculation shows that the mode shape of a straight pipe clamped on its left side and without any $\exp(kx)$ term depends on only one amplitude coefficient C:

$$y(x) = C[\exp(-kx) - \cos(kx) + \sin(kx)].$$

The coefficient C can be expressed as a function of the bending stress using the expression of the bending moment and the relation $\sigma_0 = M_0\,R_{\text{out}}/I$:

$$k^2y(x) = \frac{\sigma_0}{2R_{\text{out}}E}[\exp(-kx) - \cos(kx) + \sin(kx)].$$

The k^2 term suggests to use the dispersion relation and to replace the deflection spectrum y by the velocity spectrum v:

$$v(x) = \sqrt{\frac{EI}{\lambda}}\frac{\sigma_0}{2R_{\text{out}}E}[\exp(-kx) - \cos(kx) + \sin(kx)].$$

This expression can be rearranged in nondimensional form

$$v(x) = \alpha\frac{\sigma_0}{E}[\exp(-kx) - \cos(kx) + \sin(kx)],$$

where α is equal to $\alpha = \frac{1}{2}\sqrt{\rho_{\text{steel}}I/(\lambda R_{\text{out}}^2)}$. For steel tubes complying with the ASME B.3610 M-1996 standard, the term α can be computed as a function of the outer diameter. As can be seen in Fig. 13, α varies from 0.2 to 0.3 in a wide range of diameters.

Note: the dispersion relation can be rewritten as $k^2 = \omega/2\alpha c_{\text{steel}}R_{\text{out}}$

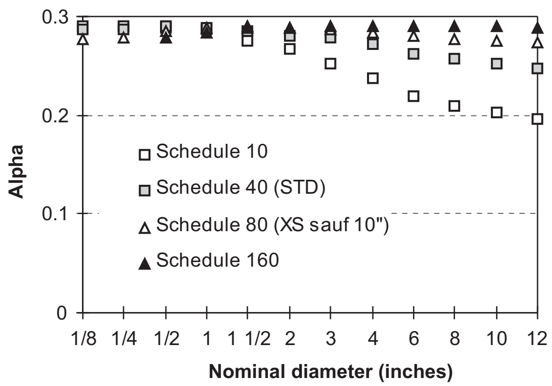

Fig. 13. Values of the alpha term for tubes complying with ASME B.3610 M-1996.

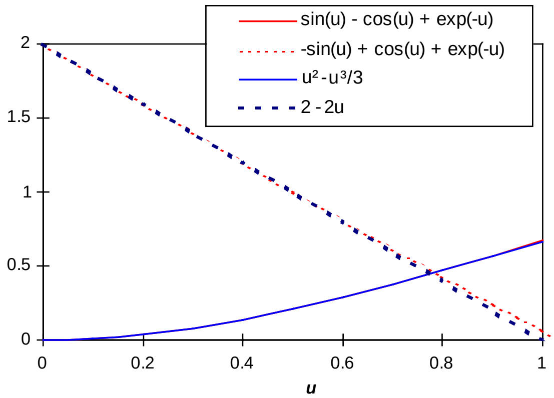

Fig. 14. Static deflection for a pure shear force at $u = 1$.

A.3. Justification of the removal of the exp(kx) term

The goal of the study is to derive a unique relation between the amplitude of the mode shape and the stress of the straight part of the pipe. Strictly speaking, the mode shape of an Euler beam exhibits four terms: a sine, a cosine and the exponential terms $\exp(kx)$ and $\exp(-kx)$. Assuming a clamp condition on the left side of the beam, two relations exist between the amplitudes of the four terms. An extra relation is then required to reduce the number of independent terms. To fulfill this requirement, it is proposed to suppress the $\exp(kx)$ term.

The underlying idea is that if the term $\exp(kx)$ had a noticeable influence on the left side of the straight pipe, it would dominate the deflection, the angle, the moment and the shear force on the right side. As a mode shape of the type $y \sim \exp(kx)$ does not seem physically acceptable, the term $\exp(kx)$ should be negligible in most practical case.

As a numerical illustration, the cantilever beam of Fig. 15 can be considered. Blevins (1984) provides the following mode shape:

$$y(x) = \cosh(kx) - \cos(kx) - 0.7341[\sinh(kx) - \sin(kx)],$$

which can be rewritten as

$$y(x) = 0.1330 \exp(kx) - \cos(kx) + 0.8670 \exp(kx) - 0.7341 \sin(kx).$$

The amplitude of the $\exp(kx)$ term is six times smaller than the other ones.

As shown in Figs. 14–16, the approximate mode shape is compared with several mode shapes given by Blevins (1984). A fair agreement is obtained in all cases, so that the removal of the exponential term is justified.

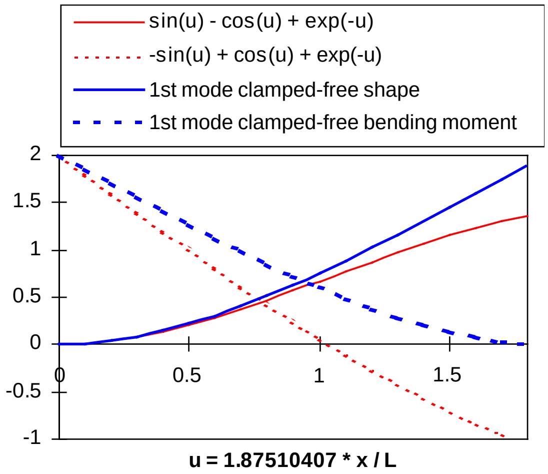

Fig. 15. Mode shape for a clamped–free beam.

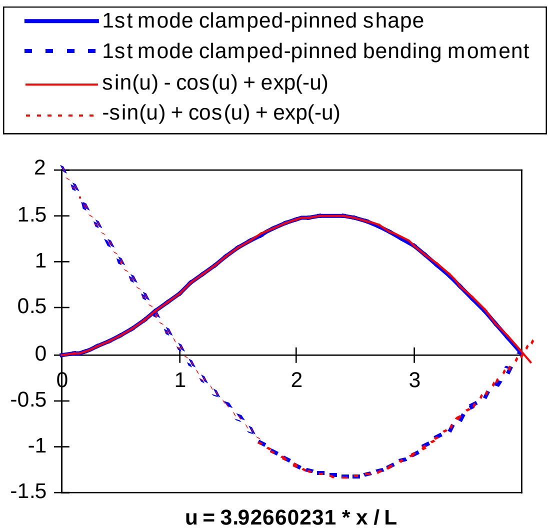

Fig. 16. Mode shape for a clamped–pinned beam.

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